Reg. No.: | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

PG DEGREE END SEMESTER EXAMINATIONS - APRIL 2025.
(For those admitted in June 2023 and later)

PROGRAMME AND BRANCH: M.Sc., MATHEMATICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
IV	PART-III	CORE ELECTIVE-6	P23MA4E6A	RING THEORY AND LATTICES

Date & Session : 29.04.2025/FN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** **Answer ALL Questions.**
CO1	K1	1.	The homomorphism φ of R into R' is an isomorphism if and only if $I(\varphi)$ is a) 0 b) 1 c) R d) all functions of R
reCO1	K2	2.	If U and V are the left and right ideals of R respectively then a) UV is not an ideal of R b) UV is a left ideal of R c) UV is a right ideal of R d) UV is a two sided ideal of R
CO2	K1	3.	If a and b are arbitrary non zero elements of a Euclidean ring of R. If $a \in R$ is a unit in R then a) $d(a,b) = 1$ b) $d(ab) = d(a)$ c) $d(ab) \leq d(a)$ d) $d(ab) \geq d(a)$
CO2	K2	4.	Which of the following Integral domain is Euclidean ring a) Ring of integers b) The Gaussian integers c) Both a) and b) d) None of the above
CO3	K1	5.	Which of the following is true? a) The polynomial ring $Z(x)$ over the ring of integers is a Principal ideal b) The polynomial ring $F(x)$ over the field F is a Principal ideal c) The polynomial ring over an arbitrary ring is a Principal ideal d) All the above
CO3	K2	6.	Which of the following is a primitive polynomial? a) $3x^2 + 6xy + 81$ b) $2x^2 + 3x + 5$ c) $4x^2 + 2x$ d) $9x^2 + 6x + 27$
CO4	K1	7.	Which of the following is modulus equation? a) $x \vee (y \wedge z) = (x \vee y) \wedge z$ b) $x \vee (y \wedge z) \leq (x \vee y) \wedge z$ c) $x \vee (y \wedge z) \geq (x \vee y) \wedge z$ d) $\overline{x \vee y} = \bar{x} \wedge \bar{y}$
CO4	K2	8.	A complemented distributive lattice is said to be a) Boolean algebra b) commutative lattice c) Bounded lattice d) associated lattice
CO5	K1	9.	A lattice L with 0 and 1 is said to be complemented if for any $a \in L$there exists a' such that a) $1 = a \vee a'$ b) $a \wedge a' = 0$ c) both a) and b) d) either a) or b)
CO5	K2	10.	A ring called Boolean if all its elements are a) nilpotent b) idempotent c) commutative d) all the above

Course Outcome	Bloom's K-level	Q. No.	**SECTION – B (5 X 5 = 25 Marks)** **Answer ALL Questions choosing either (a) or (b)**
CO1	K2	11a.	If R is a commutative ring with unit element and M is an ideal of R then M is a maximal ideal of R if and only if $\frac{R}{M}$ is a field.
			(OR)
CO1	K2	11b.	Let R be the ring of integers and U is an ideal of R, U consists of all the multiples of fixed integer n_0, that is , $U = (n_0)$. What values of n_0 lead to maximal ideals?
CO2	K2	12a.	Let R be a Euclidean ring. Then any two elements a and b in R have a greatest common divisor d. Moreover $d = \lambda a + \mu b$ for some $\lambda , \mu \in R$
			(OR)
CO2	K2	12b.	State and Prove FERMAT theorem
CO3	K3	13a.	Prove that, the ideal $A = (p(x))$ in $F[x]$ is a maximal ideal if and only if $p(x)$ is irreducible over F
			(OR)
CO3	K3	13b.	Prove that, if R is an integral domain, then so is $R[x]$
CO4	K3	14a.	Prove that in a bounded distributive lattice, the complement of any element is unique
			(OR)
CO4	K3	14b.	Prove that every distributive lattice is modular
CO5	K4	15a.	Prove that, Let $S = S_1 \times S_2$ where S_1 is partially ordered set. Let μ, μ_1, μ_2 be the mobious functions of S, S_1, S_2 respectively. Then $\mu(x_1, x_2)(y_1, y_2) = \mu_1(x_1, y_1)\, \mu_2(x_2, y_2)$
			(OR)
CO5	K4	15b.	Prove that, a lattice L is modular if and only if whenever $a \geq b \text{ and } a \wedge c = b \wedge c$ and $a \vee c = b \vee c$ for some $c \in L$ then $a = b$

Course Outcome	Bloom's K-level	Q. No	**SECTION – C (5 X 8 = 40 Marks)** **Answer ALL Questions choosing either (a) or (b)**
CO1	K4	16a.	Prove: If U is an ideal of the ring R then $\frac{R}{U}$ is a ring and is a isomorphic image of R
			(OR)
CO1	K4	16b.	Prove: Every integral domain can be imbedded in a field
CO2	K5	17a.	Prove: The ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R.
			(OR)
CO2	K5	17b.	Prove: $J[i]$ is a Euclidean ring
CO3	K5	18a.	State and prove THE DIVISION ALGORITHM
			(OR)
CO3	K5	18b.	Prove that, If R is a unique factorization domain then so is $R[x]$
CO4	K5	19a.	Prove that, the lattice of normal subgroups of a group is modular. The lattice of submodules of a module is modular
			(OR)
CO4	K5	19b.	State and prove DeMorgan's laws of lattices
CO5	K6	20a.	Prove: $(B, \vee, \wedge, 0, 1, ')$ is a Boolean algebra
			(OR)
CO5	K6	20b.	Prove: The complement a' of any element a of a b Boolean algebra B is uniquely determined. The map $a \rightarrow a'$ is an anto-automorphism of period ≤ 2: $a \rightarrow a'$ satisfies DeMoegan's law and $a'' = a$